**ISTANBUL BILGİ UNIVERSITY**

**FACULTY OF ENGINEERING AND NATURAL SCIENCES Department of Electrical-Electronics Engineering**

**EEEN 460**

**OPTIMAL CONTROL**

**MIDTERM EXAM**

**HONOR CODE**

Dear Students, we invite you to be dedicated to protect the

Integrity of this exam, as well as yours and your classmates’s work and efforts.

As a part of this dedication we ask you to read and follow the following rules.

Please

* Submit your own original work
* Avoid sharing answers with others
* Report suspected violations

Thank you for your cooperation.

**NOTE:**

**There are 8 questions in this exam. Please Note that the first and last questions are worth 20 points and the rest are 10 points. Give your answers by using this document, i.e. enter your answers to the provided blank spaces, you may add additional pages. When you complete this exam submit it to the inbox as a Word document. Inbox will be open till 23:59.**

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| Name | Surname |
| Tuğberk | Göç |

1. Write the performance measure index for each of the following cases **(20 points):**
2. A racing car runs to finish the race first

Suppose the objective is to make the car reach final point as quickly as possible; then the performance measure J is given by,

J = -

In all that follows it will be assumed that the performance of a system of a system is evaluated by a measure of the form,

J = h (x ( +

h(( + ≤ h(x( +

1. A controller tries to keep the level of oil in a container at constant level.

If the controller does not try to keep the level of oil in a container at constant level, car starts G gallons of gas and decrease the gas with time. And its formula;

(t)] dt ≤ G

But if the controller tries to keep the level of oil in container at constant level, controller should not use throttle. That’s why our formula should be,

] dt ≤ G

1. An anti-aircraft missile trying to hit an enemy aircraft.

To transfer a system from arbitrary initial state to a specified target set S in minimum time. The performance measure to be minimized is,

J = -

with the first instant of time when intersect. It is the typical example for the interception of attacking aircraft and missiles.

J = h (x( +

It can be evaluated with this formula.

1. A defensive interceptive aircraft trying to follow an intrusive enemy aircraft as closely as possible.

In this statement, we can use this equation below,

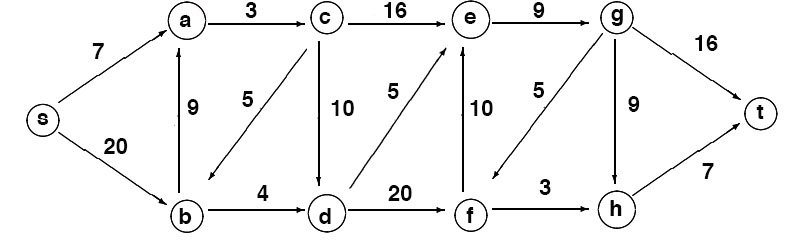
h(( + ≤ h(x( + that formula. Because a defensive interceptive aircraft trying to follow an intruvise enemy **aircraft as closely as possible**. That is, it does not hit the enemy aircraft so our h(( + could be less than 0.

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1. A driver can travel on the streets shown on this map where costs of sub-paths are given.

Find the optimum path from **start (s)** to **target (t)** by using dynamical programming

**(10 points).**

****

**North**



**East**

**South**

**West**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Current Intersection**  **α** | **Heading** | **Next Intersection** | **Minimum cost from α to h via**  = | **Minimum cost to reach h from α** | **Optimal Heading at α** |
| **h** | **North-East** | **t** | **7 + 0 = 7** | **7** | **North-East** |
| **g** | **South-East**  **South**  **South-West** | **t**  **h**  **f** | **16 + 0 = 16**  **9 + 7 = 16**  **5 + 10 = 15** | **15** | **South-West** |
| **e** | **East** | **g** | **9 + 15 = 24** | **24** | **East** |
| **f** | **North**  **East** | **e**  **h** | **10 + 24 = 34**  **3 + 7 = 10** | **10** | **East** |
| **d** | **North-East**  **East** | **e**  **f** | **5 + 24 = 29**  **20 + 10 = 30** | **29** | **North-East** |
| **c** | **East**  **South-West**  **South** | **e**  **b**  **d** | **16 + 24 = 40**  **5 + J\*cb = 5 + x**  **10 + 29 = 39** | **m** |  |
| **b** | **North**  **East** | **a**  **d** | **9 + J\*ba= 12 + m**  **4 + 29 = 33** | **x** |  |
| **a** | **East** | **c** | **3 + J\*ac = 3+m** | **3+m** |  |
| **s** | **North-East**  **South-East** | **a**  **b** | **7 + J\*sa =**  **20 + J\*sb =** |  |  |

**When we look at the minimum cost from [c to h] and [b to h];**

**min(x+5, 39, 40) = m**

**min(12+m, 33) = x**

**x + 5 m**

**Hence, x will be 33 because I’ve selected second equation (min(12+m, 33) = x) as x equals to 33. In this situation, m will be 38 and this solution satisfy all equations.**

**When I have tried other option, which is m = 39, we found x equals to 51. This solution does not satisfy in second equation (min(12+m, 33) = x).**

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| **Current Intersection**  **α** | **Heading** | **Next Intersection** | **Minimum cost from α to h via**  = | **Minimum cost to reach h from α** | **Optimal Heading at α** |
| **h** | **North-East** | **t** | **7 + 0 = 7** | **7** | **North-East** |
| **g** | **South-East**  **South**  **South-West** | **t**  **h**  **f** | **16 + 0 = 16**  **9 + 7 = 16**  **5 + 10 = 15** | **15** | **South-West** |
| **e** | **East** | **g** | **9 + 15 = 24** | **24** | **East** |
| **f** | **North**  **East** | **e**  **h** | **10 + 24 = 34**  **3 + 7 = 10** | **10** | **East** |
| **d** | **North-East**  **East** | **e**  **f** | **5 + 24 = 29**  **20 + 10 = 30** | **29** | **North-East** |
| **c** | **East**  **South-West**  **South** | **e**  **b**  **d** | **16 + 24 = 40**  **5 + 33 = 38**  **10 + 29 = 39** | **38** | **South-West** |
| **b** | **North**  **East** | **a**  **d** | **9 + 41 = 50**  **4 + 29 = 33** | **33** | **East** |
| **a** | **East** | **c** | **3 + 38 = 41** | **41** | **East** |
| **s** | **North-East**  **South-East** | **a**  **b** | **7 + 41 = 48**  **20 + 33 = 53** | **48** | **North-East** |

PATH: s 🡪 a 🡪 c 🡪 b 🡪 d 🡪 e 🡪 g 🡪 f 🡪 h 🡪 t

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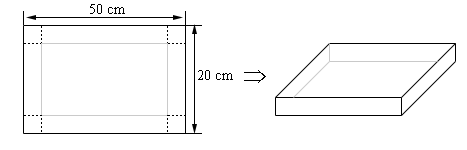
1. We have a piece of cardboard that is 50 cm by 20 cm, and we are going to cut out the corners and fold up the sides to form a box.

a) Show that the largest value the height (h) could be ½ times the smaller side (3 points).

b) Determine the height (h) of the box that will give a maximum volume (5 points).

c) What is the maximum possible volume?

**(10 points)**



x

50-2x

x

x

20-2x

x

Volume of the box = x b x h

=

Thus,

1. Now, for max volume,

Now,

When;

x = 18.9315 =>

x = 4.402 =>

Hence,

Volume is maximum at x = 4.402cm

length = 50-2x = 41.196 cm

breadth = 20-2x = 11.196 cm

And,

Volume = 0 for x = 0, 10, 25.

‘x’ should be greater than ‘0’ and ‘x’ should be 10 and ‘x’ should be 25

x(0,10)

Hence, maximum value of height = 10 which is ½ \* [20].

Max value of height(x) = ½ \* smallest side (b)

1. As calculated in (a), for max volume

x = 4.402 cm = h

1. Max volume = [From a]

= (41.196) x (11.196) x (4.402)

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1. In the system given below (**10 points):**

Objective function is given by

**p (x, y) =150x+50y**

Maximize it by using the graphical method,

under the constraints:

**4x+2y≤800**

**2x+y≤700**

**0≤x≤100**

**0≤y​≤400**

1. What is **(x\*, y\*)?**
2. What is **p\*(x\*, y\*)?**

4x + 2y = 800 Points 🡪 for x=0 (0, 400)

🡪For y=0 (200, 0)

2x + y = 700 Points 🡪 for x=0 (0, 700)

🡪 for y=0 (350, 0)

**x≤100**

700

(100,400)

**y​≤400**

400

x = 100

4x + 2y = 800

2y = 400, y = 200

(100, 200)

200

100

350

**2x+y≤700**

**4x+2y≤800**

(x, y) P(x, y)

(0,400) 20000

(100,400) 35000 (MAX)

(100,200) 25000

b)

P\*(x\*, y\*) = 150\*(100) +50\*(400)

= 35000

a)

X\*=100

Y\*=400

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1. **By using Simplex method maximize (10 points):**

P = 7x + 12y

**Standart Form**

P-7x-12y=0

2x+3y+S1=6

3x+7y+S2=12

**Subject to:**

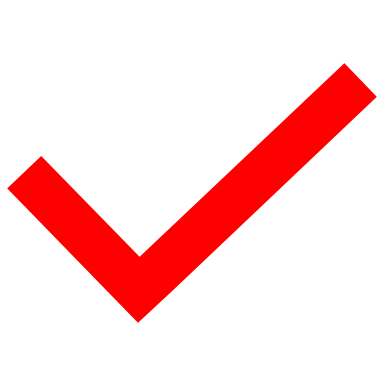
2x + 3y ≤ 6  
3x + 7y ≤ 12

x≥0

y​≥0

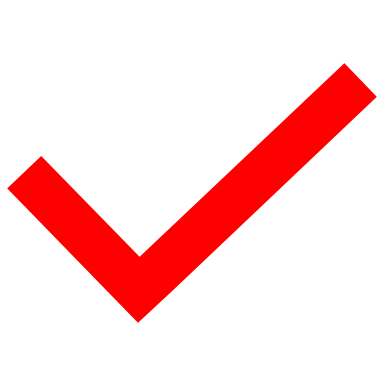
MRT

6/3 = 2

12/7 = 1,71 

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | P | x | y | S1 | S2 |  |
| P | 1 | -7 | -12 | 0 | 0 | 0 |
| S1 | 0 | 2 | 3 | 1 | 0 | 6 |
| S2 | 0 | 3 | 7 | 0 | 1 | 12 |

MRT

6/7\*7/5 = 6/5 = 1,2 

12/7\*7/3 = 3

R3🡪R3/7

R2🡪R2-(3/7)R3

R1🡪R1+(12/7)R3

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | P | x | y | S1 | S2 |  |
| P | 1 | -13/7 | 0 | 0 | 12/7 | 144/7 |
| S1 | 0 | 5/7 | 0 | 1 | -3/7 | 6/7 |
| y | 0 | 3/7 | 1 | 0 | 1/7 | 17/7 |

In order to not to have non-negative values in R1 row(colored as yellow above), I’ve done another MRT as you can see below.

R3🡪R3-(3/5)\*R2

R2🡪R2\*(7/5)

R1🡪R1+(13/5)R2

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | P | x | y | S1 | S2 |  |
| P | 1 | 0 | 0 | 13/5 | 21/35 | 642/35 |
| X | 0 | 1 | 0 | 7/5 | -3/5 | 6/5 |
| y | 0 | 0 | 1 | -3/5 | 9/35 | 42/35 |

Now, we are ready to get x, y and p optimized values. Also, including s1 and s2.

Inactive Nodes

S1 = 0

S2 = 0

Active Nodes

P\*= = 18,34

X\*= = 1,2

Y\*= = 1,2

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1. Consider a mass m on the end of a spring of natural length l and spring constant ***k***. Let ***y*** be the vertical coordinate of the mass as measured from the top of the spring. Assume the mass can only move up and down in the vertical direction. The Lagrangian (***L***) is given for this system as



1. Write the corresponding Euler-Lagrange equation.
2. Solve this equation to obtain ***y***

**(10 points)**

Euler Langrange equation is;

, then equation becomes,

Auxiliary Equation of DE is;

Particular Solution is,

so put

Thus, final equation has been as it shown below,

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1. Using the Euler –Lagrange equation **(10 points)**

a) Find y(x) (as an equation) which minimizes the following objective function



b) Determine the parameters in a)

if a = (-3, 8), and b = (3, -6)

a) Think it straight line:

Assume that

=

Using Euler-Lagrange Equation:

and

and =0

The optimal solution is a straight line :

b)Find the solution when;

a = (,

b = (, ) = (3, -6)

With respect to and

8 = -3+ (1)

-6 = 3+ (2)

According to (1) and (2): and

The optimum solution is a straight line:

|  |  |
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1. The first order discrete system **(20 points)**

x(k+1)=0.5x(k)+u(k)

is to be transferred from initial state x(0)=-2 to final state x(2)=0

in two states while the performance index

is minimized.

Assume that the admissible control values are only

-1, 0.5, 0, 0.5, 1

Find the optimal control sequence

and also so,